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ABSTRACT

This focuses on movements to asses learning what is basic at various levels: movements to align curriculum to standards at all levels, and familial pressures to guarantee that children's mathematics education includes the mathematics of the future. This newsletter provides 15 articles on the above theme: (1) "What's Basic in Math Education: A View from Mathematically Correct;" (2) "Forward, with Caution, to (the New) Basics;" (3) "Basic Mathematics and the Public Perception;" (4) "The Basics: More Than Rote Learning of Basic Facts;" (5) "The Real Basics Are Children;" (6) "Middle School Basics;" (7) "Basic Betrayals: Middle School Mathematics;" (8) "Basic Skills Instruction is the Key to a Happy Marriage;" (9) "A Pythagorean View of the Basics;" (10) "Readers Respond to What's Basic;" (11) "Basic Skills Should Include Mental Computation;" (12) "Narrowing the Curriculum River;" (13) "A National Curriculum?;" (14) "Our Readers Count on Calculators;" and (15) "Questionnaire." (CCM)

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Mathematics Education

Dialogues

Back to the Basics or Forward to the Basics—Which Philosophy Should We Embrace?

Inside this issue . . .

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The basics of school mathematics concern many, as evidenced by movements to assess learning what is basic at various levels, by movements to align curriculum to standards at all levels, and by familial pressures to guarantee that children's mathematics education includes the mathematics needed for the future.

According to Pilarski, in *Mathematics Education: Wellspring of U.S. Industrial Strength* (Marriott Hotels 1989)—

> Today's worldwide, computer-driven, competitive economy demands workers with thinking skills, workers who can deal with computer terminals, automated equipment, and visual data display; who can make estimates and solve problems; who have a mental "toolkit" of number-managing techniques. The rudimentary skills that satisfied the needs of the workplace in the past no longer suffice.... But the standard mathematics curriculum still presents, virtually unchanged, a body of doctrines dating back to the 15th century.

The absence of a national curriculum in both Canada and the United States has led to provincial agreements in Canada and to some state-adopted curricula that make decisions about what is basic.

The reform movement in mathematics education and reaction to it led the Minnesota Council of Teachers of Mathematics to request that *Dialogues* initiate a discussion of basics. Anne Bartel, president of the Minnesota Council, wrote—

> We would benefit from a *Dialogues* issue focused on basic skills in standards-based mathematics curriculum and instruction. Questions might include these:
> - What are "basic skills" in an age of calculator technology?
> - When should students know and be able to perform them?
> - What is the balance between concepts and procedures in a standards-based curriculum?

The Editorial Panel of *Dialogues* sought authors who could set the stage for a balanced debate—not just on basics for standards-based mathematics education but also on basics in a technological age. We invite your responses.

by Johnny W. Lott, Editor
For the Editorial Panel

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What's Basic in Math Education:

A View from Mathematically Correct

by Martha Schwartz

Mathematically Correct is about back-to-basics. Right? More "correctly," we are for good, balanced mathematics education, but the basics are indeed an important part of the story. I can't provide an exhaustive list of what's basic at every level, but I can recognize when something basic is missing. I recognized it when I put two mixed numbers, such as 3 7/8 and 5 1/4, on the chalkboard the first day of my college remedial algebra class. I told the students that their two tasks were to add the numbers and then multiply them. Only three students out of about fifty claimed comfort with this request. We compiled suggested answers, a few of which were way off the mark. I asked whether anyone could think of a way to eliminate some of the incorrect answers before performing the calculations. Nobody could. Two basics were missing—the ability to perform simple arithmetic computations and the insight to use estimation as a check of reasonableness. Learning algebra was very hard for these students.

Basics are those fundamental building blocks on which later learning often rests. They are, as well, the movable blocks that can be grabbed and put together in creative ways to solve problems. They are not always the most interesting mathematics, the most fun, the most beautiful, or even the most immediately useful. But they are absolutely essential.

Basics are definitely not just those utilitarian skills that people need to plow through everyday life and commerce, although these are coincidentally a subset. Basics also include the pieces that a student must have in place to tackle technical subjects, should he or she eventually decide to do so.

At least until tenth grade, basics are the same for all students. It is presumptuous to decide in early childhood who is college bound and who is not, who might someday decide to become a mathematician, scientist, physician, or engineer...; everyone deserves solid preparation for a full set of choices.

What is basic changes with the age of the child. In the very young, counting is basic—reciting numbers—but also basic is the deep understanding that counting objects means putting them in one-to-one correspondence with those numbers.

In the elementary grades, whole-number arithmetic is basic—the ability to perform computations routinely, to understand why algorithms work, and to know how to apply algorithms properly to simple real-life and abstract problems.

Understanding rational numbers—fractions, decimal and percent equivalents—is basic. Middle-grade students should calculate with them; think with them; understand, for example, why, in a percent problem, the base is just as important as the rate.

Much of beginning algebra is likewise basic. Not just the manipulations but the magical insight that anyone can assign a variable to represent an unknown quantity, write an equation, and solve it—and that doing so makes complicated-looking puzzles routine.

The basics aren't everything. In isolation, they make a deadly dull curriculum. But when they're missing, it's terribly hard to make up later for the loss. And as my students now know, their absence slams doors; I hope not always permanently.

Martha Schwartz is a lifelong learner, having received a B.S. (mathematics, Arizona State) in 1966, M.S. (geology, CSU Long Beach) in 1984; and Ph.D. (geophysics, University of Southern California) in 1997. She currently does paleomagnetic research at the University of Southern California; teaches classes at CSU, Dominguez Hills; and is a cofounder of Mathematically Correct.

The purpose of ***Mathematics Education Dialogues*** is to provide a forum through which NCTM members can be well informed about compelling, complex, timely issues that transcend grade levels in mathematics education. The opinions expressed in this publication are those of the writers and do not constitute an official position of the NCTM. ***Mathematics Education Dialogues*** is published as a supplement to the ***News Bulletin*** by the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-9988. Pages may be reproduced for classroom use without permission.

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FORWARD, with Caution, to (the New) Basics

by Keith Devlin

"Back to basics!" is a familiar cry. What those crying usually mean is a reemphasis on fundamental arithmetic skills. The reason given is that, more than ever, the world requires a populace skilled in arithmetic.

In giving this reason, the back-to-basics brigade is wrong on at least two counts. First, living in today's world does not require that everyone must have arithmetic ability. Second, even if it did, considerable evidence suggests that such an ability cannot be taught in a school setting.

However, because the back-to-basics movement might base its proposal on a wrong reason does not automatically mean that the conclusion is wrong. To set the scene for the "basics," realize that teaching mathematics in school first arose in Britain around 1750, when arithmetic was brought into elementary schools from the marketplace. The mathematics curriculum then consisted of memorizing and using the monetary system and traders' systems of weights and measures. American schools followed Britain's lead for the same reason: to teach people the basic systems of measurement needed in everyday lives.

Arithmetic came later. The curriculum evolved to be less mercantile and more "academic" until about 1820, when school mathematics was basically learning the rules of arithmetic with few connections to the marketplace. The assumption was that if pupils were taught to master general procedures for computations, they could apply them in any situation. However, evidence shows that many people are unable to apply in real-world contexts the arithmetic procedures that they learn *in the classroom*. When faced with calculations in a real context, most people generally develop their own methods. See, for example, Jean Lave's *Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life* (Cambridge: Cambridge University Press, 1988) and *Street Mathematics and School Mathematics*, by Terezinha Nunes, Analucia Schliemann, and David Carraher (Cambridge: Cambridge University Press, 1993).

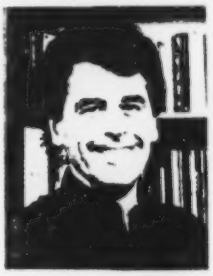
Today, we have cheap and readily available machines that do arithmetic. Today's citizen no more needs to know how to add, subtract, multiply, or perform long division than to be able to plow or to ride a horse.

However, those machines have helped create a society in which an informed citizen needs greater mathematical skills—a new set of basics—than ever before. "Basic math" for the information age involves abstract patterns, relationships, and structures, as well as numbers. Consequently, basic mathematics skills for today's students are not the same as those that their parents learned. Contrary to newspaper articles, that fact does not make the mathematics easier or less rigorous. Quite the opposite. For example, using calculators and computers properly does not represent a reduction in skill or need for accuracy but requires far greater skill, and much more mathematical insight, than we old-timers had to master to get our sums right.

So the back-to-basics movement has it all wrong when it comes to the reason put forward for reemphasizing mastery of basic arithmetic skills. Today's essential "basics" are quite different from those of the past.

And yet, it is possible that teaching basic arithmetic, as well as the new basics, is still important. The back-to-basics advocates might be partially right for the wrong reason. We do not have a sufficient understanding of the way the brain works—and how people learn—to know how much arithmetic drill is necessary to prepare a mind to acquire the new basics. Just because mental arithmetic is no longer an important life skill in itself does not automatically mean that the *process* of learning how to do arithmetic produces no benefit. Indeed, *going through that process* may be an important precursor to learning the skills that are now important. My argument is not that all mathematically able individuals in the past went through such a process—although that statement is true and simply reminds us that we have no evidence to indicate that mathematical skills can be acquired without such a precursor. Rather, I base my question on what little we do know about the way that the brain works, in particular, on how it copes with abstraction.

Is arithmetic drill still important? I don't know. More to the point, nor do any of the combatants in the current "math wars." If ever a time was right for more research and less rhetoric and dogma, this is it. When ignorant of the consequences, it is best to proceed with caution.



Keith Devlin is Dean of Science at Saint Mary's College of California and a senior researcher at Stanford University. He was a lead adviser on the 1998 PBS television series Life by the Numbers *and author of the companion book of the same title, both of which set out to demonstrate the importance of the "new basics" to which he alludes in his article.*

Basic Mathematics and the Public Perception

by Richard C. Cole

From my experience, public perception about mathematics basically is at best confused and at worst nonexistent. Moreover, somewhere between best and worst is a large dose of misperception fueled by the fact that arithmetic was the mathematics taught to the majority of the general public during the 1950s, 1960s, 1970s, and into the early 1980s.

Realize, however, that my frame of reference is limited to Connecticut. The public perception here is that children should know and be able to manipulate numbers in their heads and carry out arithmetic operations accurately, first without calculators and then with calculators. Finally, don't even consider leaving out a thorough understanding of multiplication tables.

From my perspective, the general public doesn't fully appreciate the difference between arithmetic and mathematics. Furthermore, the intellectual discussion among mathematicians and mathematics educators about basics instruction or hands-on, minds-on, challenging integrated mathematics courses does nothing to dispel the confusion. Then by adding a few rubrics, we truly have a dysfunctional tête-à-tête. Remember that I'm talking about public perception, not what the best method is to teach mathematics.

The difference between arithmetic and mathematics can be described by using foreign-language learning as an analogy. When studying a new language, the first things encountered are long lists of words to learn and memorize, when appropriate. The learner must understand and "own" basic grammatical concepts, facts, and verb conjugations. These basics are analogous to arithmetic in learning mathematics.

However, for the language to be useful, an understanding of the words and how they work together to create meaning—the appreciation that manifests itself when using a language appropriately in a contextual sense—is necessary. Likewise, going beyond arithmetic opens up the wonderful world of mathematics, a language of logic and precision. I use this analogy with the public because it works.

I am distressed that almost every person with whom I speak believes that few students are capable of doing mathematics well. However, student-performance data from the standards-based Connecticut Mastery Tests demonstrate that more students are acquiring more skills than ever before.

Special-interest groups aside, I find that public perception about what mathematics education should be is simply to "prepare children to have the necessary skills to make meaningful life choices upon graduation." The burning public issue is not "basics" rather than "hands-on"; it is "prepare my child for tomorrow."

Richard Cole is the executive director and chief executive officer of the Connecticut Academy for Education in Mathematics, Science and Technology in Middletown, Connecticut. The Academy is responsible for Project CONNSTRUCT, a project funded by the National Science Foundation as a Statewide Systemic Initiative.

The Basics: More Than Rote Learning of Basic Facts!

by Jacqueline Goodloe

Recently, a second-grade teacher asked me, "Do children really need to be taught basic facts, or do they just know them?" I politely responded, "Yes, children do indeed need to be taught basic facts, and they also need to be presented with various strategies to learn them." After thinking more about this question from a seasoned veteran, I observed her in the classroom. She taught basic facts by presenting addition and subtraction facts and then requiring that students memorize them through practice and drill. Her teaching revealed her belief that failure to learn basic facts resulted from failure to sit down and memorize them.

A question facing many such teachers is, If I de-emphasize drill and practice as the NCTM's *Standards* suggests, what is my responsibility as the teacher? What do I emphasize? My answer is, Emphasize thinking! We must teach our children thinking strategies and techniques that enable them to recall addition, subtraction, multiplication, and division facts. We must also challenge them with problem situations so that they see the need to have these facts readily available.

Parents of preschoolers are proud when their young prodigies enter school able to count to 100. Very few can identify the number that comes after 100, suggesting that they have learned to count purely by rote. But rote memorization should be tied to conceptual understanding, which in turn requires that children engage in counting activities in a variety of situations over a prolonged period of time.

Building on the experiences that children bring to school is where teaching basic skills begins. Providing rich, sound, connected mathematics helps students develop higher-order thinking. However, basic mathematics skills involve more than just knowing the arithmetic facts for the first ten whole numbers. In particular, children should have a sense of space and shape; a sense of number, number meanings, and number operations; and a sense of making appropriate measurement estimations. Children should acquire these understandings, as well as fact skills, as a part of a process of developing a sound mathematical basis for higher-level thinking.

Jacqueline Goodloe is an elementary mathematics resource teacher in the District of Columbia Public Schools. She is a 1991 Presidential Awardee in elementary mathematics.

The Real Basics Are *Children!*

by Vicki Walker

What are the basics in school mathematics? should be replaced with, Why are we so consumed with this focus on the basics? Why do we spend so much time debating what the basic skills are, how they should be taught, and how much instructional time should be allocated to mastering them? What's the real motive behind the basics question?

The motive may be that we, as teachers, cater to a "recipe" mentality toward mathematics education. We want a quick and easy fix with a list of ingredients that clearly delineates what is important. We want step-by-step directions telling us what to do, how to blend things, what the specific yield will be, and precisely how long it will take. We want a "test for doneness" with immediate, tangible results. We are comforted by the security of having a recipe, and we rely on older, fail-safe methods.

In doing so, we give too little consideration to whom we are serving and too much consideration to what we are accomplishing. We focus heavily on the end result and focus minimally on the process that occurs along the way. We discount students' ideas and conceptions so that we can cover material and stay on schedule. We dole out information to students and contend that we are giving them a well-balanced mathematical diet. We insist that quick mastery of paper-and-pencil computations is a prerequisite to higher-level mathematical thinking. We instruct students the way we concoct culinary creations, as if we are the chefs and our students are our creations. We are driven by what textbooks say, test scores show, and schedules allow. In our pursuit of the basics, we have lost sight of the single most basic ingredient within our mathematics classrooms, *the student*.

The real questions should be these: How can we turn our attention back to the children? How can we redirect our conversations so that we center our thoughts more on children and less on content? When will we recognize that the present recipe mentality toward mathematics education lacks nutritional value for our students? When will we be brave enough to make educational decisions because they are right for the children, not just because they are quick and easy for teachers, parents, and administrators? And finally, when will the question regarding basics in school mathematics inspire a genuine dialogue about the basic mathematical welfare of our children?

Vicki Walker is an instructor for preservice elementary teachers in mathematics education at Indiana University Purdue University at Indianapolis (IUPUI) and conducts staff development activities for elementary teachers in the Indianapolis Public School system. She taught mathematics for ten years in Louisville, Kentucky, at both the elementary and middle school levels. Her interests include the development of children's mathematical thinking and understanding.

Middle School Basics

by Stephen Hake

Middle school mathematics should be practical, broad-based, and attuned to the developing cognitive ability of students. These three aspects of curriculum comprise the basics of middle school mathematics: context that applies to daily lives, content that is broad in scope, and conceptual emphasis that is appropriate and necessary for future mathematical success.

Middle school mathematics should be practical. *Mathematics: Making a Living, Making a Life* (NCTM 1996) emphasizes the need to prepare students for the world outside the classroom, the world of career and of daily life. In particular, the mathematics common to daily life provides contexts for the middle school curriculum. Virtually all quantitative demands on the daily lives of adults outside their careers fall within the capabilities of middle school students. For example, consider wise consumerism. From subtleties of credit card offers, to enticements of lottery advertising, to sales representations at the local mall, the mathematics required to unravel these inducements is accessible to middle school students who have a solid foundation of fraction, decimal, and percent concepts.

Basic middle school mathematics should also be broad in scope; it should display the wide spectrum of content that can be covered. Breadth of content provides a basis on which to build the mathematics needed for careers. Unlike traditional high school courses, which are often taught in confined strands, middle school mathematics helps prepare the conceptual foundations for secondary mathematics. Former students of mine have returned to middle school stating that they had already learned in middle school the mathematics that they were taught in high school. These students were familiar with the content that was being extended in their high school years. As mathematics educators weigh the potential benefits of algebra for all in the middle school, we also ought to consider the costs of narrowing the scope of the mathematics curriculum. An algebra course that includes a wider range of content than traditional algebra programs and that emphasizes practicality may allow us to accelerate the curriculum without sacrificing the benefits of practical and broad-based middle school mathematics.

Middle school mathematics also should be attuned to students' conceptual development. For instance, proportional thinking is an emerging concept that should be fostered and used to solve a wide range of practical problems. Students who develop the ability to recognize proportionality improve their grasp of the mathematics of daily life and increase the likelihood of their success in the mathematics of careers. Also, as students develop from thinking concretely to thinking abstractly, they become capable of shifting from specific examples to general applications, from measuring directly to measuring indirectly, from analyzing static events to exploring dynamic events. Nurturing the developing potential of middle school students in an ambitious but caring classroom atmosphere can produce well-trained students who competently deal with the mathematics of daily life and confidently approach the mathematics of high school and beyond.

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Stephen Hake has taught mathematics in El Monte, California, for 17 years, including 12 years at the middle school level. He is a coauthor of four textbooks published by Saxon Publishers.

Basic Betrayals: Middle School Mathematics

by Sharon Friesen, Pat Clifford, and David Jardine

"Back to basics" not only drives reactionary school reform movements (e.g., Berk [1985]; Holt [1996]) but also leads to critiques or defenses of "liberal" education (e.g., Lazere [1992]; Grumet [1993]). The idea of "the basics" underwrites the conception of curriculum guides, the development and delivery of content knowledge, an understanding of classroom work, the role of children in a classroom, the expectation of teachers' knowledge and actions, and the assessment of teachers' and children's work. Even more subtle and powerful is the use of "basics" as an incendiary clarion in public debate (Freedman 1993).

Mathematics is often driven by an analytic idea of "basicness" from the empirical sciences that states, What is most basic to any discipline is its smallest, most clearly testable bits and pieces. Such features as "math facts" are considered more basic than the complex relations that knit together numeracy, algorithms, and the operational character of mathematical work. Ironically, fragmented and isolated "basics" are abstract outcomes of a complex, theoretical process. Abstract, arcane, and analytical products are taught to young children because these facts are considered "basic." Thus, middle school teachers who complain, "How can I get on to anything good in the curriculum when these kids don't even know their basic math facts?" may dismiss the relevance of a topic like Fibonacci numbers by saying, "Well, your kids can do that, but mine aren't near that stage yet. They still can't do long division." Such attitudes mirror the textbooks' teaching of proportion, ratio, scale, equivalent fractions, and decimals as isolated fragments rather than as closely woven kin.

Substituting a misleading idea of what is basic for quality classroom work can involve a "betrayal" (Caputo 1987) of the richness of exploring the realities of the disciplines. Classrooms should explore how "the basics" of a discipline are those things that make it a living field (Snyder 1990). Education should draw children into the whole discipline. Fragmentation and isolation of "skills" may be requirements for the day-to-day work of schooling or for dealing with specific difficulties of particular children, but judging the appropriateness of such fragmentation requires understanding something more basic than such fragments. This judgment requires knowing the living discipline within which such fragments have place and meaning. Only in this wider, more difficult, and ambiguous realm of a "living discipline" does a sound pedagogical judgment emerge.

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Basic Skills Instruction Is the Key to a Happy Marriage

by Carl Burnside

Basic skills instruction in mathematics in the age of technology has been clouded with controversy. Early concrete, conceptual work by students is important in laying a foundation for their later abstract thinking. One conception of mathematics is that it is a body of rules and procedures to be performed quickly and accurately. Students are also expected to apply their computational skills—addition; subtraction; multiplication; and division; as well as using fractions, percents, and certain other basic everyday-life mathematics—in problem solving. In thinking skills, simple associations—number facts—are distinguished from complex procedures—algorithms—in which a fixed sequence of operations is performed.

Operations and field properties represent the base of higher mathematics. Calculators and computers may assist in investigating mathematical relationships at a higher level, but when technology is used, the importance of these properties is sometimes cast aside. Technology should enable students to compare the symbolic and graphical representations of functions. However, such a comparison is impossible if students do not understand the properties of, relations on, and operations with real numbers. Needed for understanding are such number properties as density; such relational properties as the reflexive, symmetric, and transitive properties; such operational properties as closure; and the commutative, associative, identity, and distributive properties.

Students must learn, apply, and synthesize these properties to fully maximize available technology. These properties are vital in such topics as functions, statistics, patterns, measurement, and geometry. For example, to study functions, students must be proficient with graphs, tables, and formulas. To become proficient, they must be familiar with the real-number system and its properties and operations.

Students must know not only how to add, subtract, multiply, and divide but also when and why each is appropriate. The biggest barrier that teachers face in teaching mathematics is students' inability to engage in problem solving and critical thinking. Students' struggles in these areas can partially be attributed to the failure of teachers to teach the "basics," that is, the facts, theories, and operations needed to understand and carry out problem-solving strategies. Students' success in problem solving and critical thinking is enhanced by a greater understanding of the fundamental properties of mathematics.

Even if students progress beyond algebra, many of their later struggles with such concepts as limits in calculus are caused by their lack of basic skills and lack of knowledge of properties. Students should not be taught basic skills and properties as isolated topics independent of technology. Students must go beyond rote identifying and explaining to using their knowledge flexibly and powerfully when confronted with unfamiliar situations requiring interpretation and real understanding. They must see basic skills and operations not as "stuff you do" but as methods for observing and organizing data. If a knowledge of skills and operations is helpful for learning mathematics, then a marriage between technology and mathematics will be a powerful one that can enhance both problem solving and critical thinking.

Carl Burnside is principal of the Dunbar High Magnet School for Science, Technology, Environment, and Mathematics (STEM) in Fort Myers, Florida. He was a high school mathematics teacher, the 1992 Florida Presidential Awardee for Mathematics, and coach of the Cypress Lake High math team that won three consecutive Mu Alpha Theta state mathematics championships and finished in the top three in the nation for three consecutive years.

A Pythagorean View of the Basics

by Judy Snyder

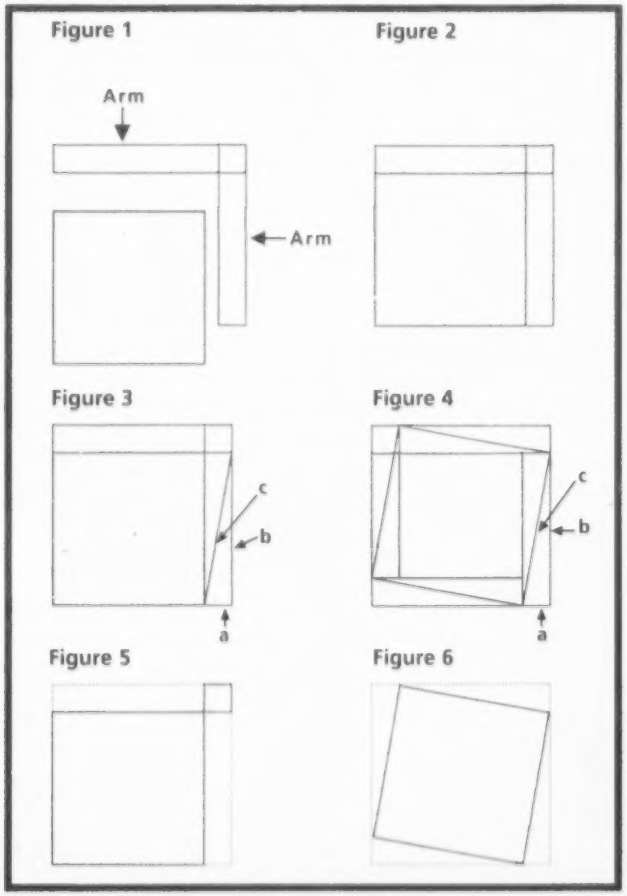
Each year I face curriculum decisions about the basics for my geometry class. Do we have time for fractals and tessellations? What must I leave out if I include some "newer" topics? Although curriculum is very important, I do not think that it is the "basic" issue.

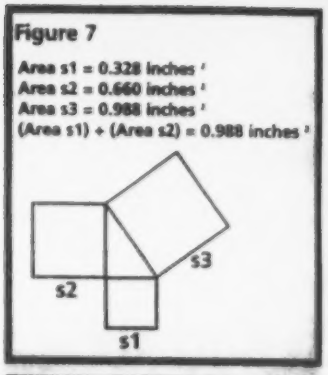
Curriculum "covered" is not necessarily curriculum learned; facts and formulas memorized are not easily remembered, much less applied. The traditional approach to mathematics—as a collection of facts, rules, and formulas to be practiced and memorized—just doesn't work. This fact is evident each time I meet parents, who invariably warn me that they cannot help their children with geometry because they don't remember it or they never really understood it.

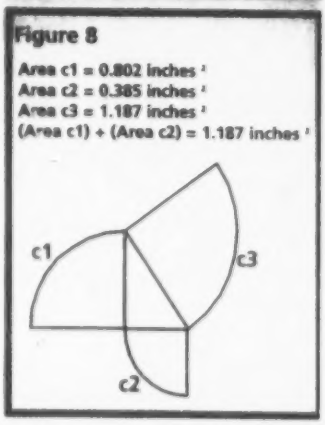
I would like students to take from my classes the ability to think mathematically to solve problems. Real problems are not always clearly stated. Students may need to sift through enormous amounts of information to find what is relevant to a problem. To become mathematical thinkers, they must develop the ability to (1) analyze as a mathematician does, (2) visualize and find patterns, (3) make and test conjectures, (4) experiment and guess, and (5) describe mathematics orally and in writing.

The following example illustrates thought processes similar to those of Pythagoras (A. K. Dewdney, *A Mathematical Mystery Tour* [New York: John Wiley & Sons, 1999]).

(1) Begin with a square and a carpenter's square with arms equal in length to the sides of the square (see **fig. 1**). (2) Attach the carpenter's square to the square, creating a larger square (see **fig. 2**). (3) Construct a diagonal in one of the arms, forming right triangles (see **fig. 3**). (4) Copy this arm three times on **figure 3**, creating **figure 4**. (5) Remove the arms from **figure 2**, creating **figure 5**; remove the outer triangles from **figure 4**, creating **figure 6**. (6) **Figures 5** and **6** have the same area. By using the lengths of the legs, a and b, and the hypotenuse c of the right triangle to compare the areas, we find $a^2 + b^2 = c^2$.







After discussing and analyzing our work, we may visualize $a^2 + b^2 = c^2$ as **figure 7** from The Geometer's Sketchpad. Since squares are similar figures, a natural question is, Do other similar figures built on the sides of right triangles produce the same results? With much discussion in small groups, we produce results similar to those in **figure 8**. (The measurements depict the original work and not the sizes of **figures 7 and 8**.)

Students then design their own explorations, making and testing conjectures, after which they write about what they have discovered and explain what they have learned.

In my opinion, students doing this exercise are learning what is basic in mathematics and are becoming mathematical thinkers. It's also possible that after such an exercise, they will remember the Pythagorean theorem.

Judy Snyder has taught at Travelers Rest High School in Travelers Rest, South Carolina, for 20 years. She currently teaches geometry, second-year algebra, and math tech. Her professional interests include interdisciplinary teaching, teaching with technology, and the development of teacher interns.

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Readers Respond to What's Basic

by Cynthia Ballheim, *For the Editorial Panel*

In the April issue of *Dialogues*, we asked readers to indicate the topics that they considered basic in seven content areas: arithmetic, measurement, estimation, algebra and functions, geometry, probability and statistics, and problem-solving strategies. Skills are basic when they are deemed necessary for later mathematics or when they are deemed so important that everyone should learn and be tested on them.

More than 540 respondents expressed a variety of opinions. The algebra-and-functions strand was the only one in which some readers (2%) declared that they would not consider any of the listed skills to be basic. In the problem-solving strand, 97 percent agreed that all the listed skills were basic.

The following results indicate the skills that readers consider basic:

In arithmetic

- Knowing when to use an operation (99%)
- Memorizing whole-number facts (99%)
- Reducing a fraction to lowest terms (92%)
- Whole-number computation using calculators (89%)
- Finding square roots with a calculator (88%)

In measurement

- Using tools to measure length (98%) and angles (89%)
- Converting units within a system (88%)

In estimation

Both skills in this area—using rounding rules to check computation and determining errors by using estimates—were considered basic by more than 80 percent of respondents.

In algebra and functions

- Solving linear equations by hand (91%)
- Multiplying binomials (82%)
- Using the quadratic formula (80%)

In geometry

- Using formulas to find area and volume of common shapes (96%)
- Applying the Pythagorean theorem (93%)

In probability and statistics

- Determining simple probabilities (97%)
- Determining mean, median, and mode (94%)

Skills that are not considered to be basic by the majority of readers included the following:

In arithmetic

- Calculating square roots by hand (79%)
- Paper-and-pencil computation with three-digit decimals (63%)

In geometry

- Doing triangle congruence proofs (65%)

In addition, Robert Reys and Richard Oldrieve responded to this survey with the two essays that follow.

Basic Skills Should Include Mental Computation

by Robert E. Reys

The questionnaire in the April 1999 *Dialogues* included a number of mathematical topics, but conspicuous was the absence of mental computation. Mental computation is an important skill for all students to possess and is a natural stepping-stone to developing written computation and computational estimation. Research shows that mental computation is used more often than written computation in out-of-school work by students and in adult daily living.

In addition to its practicality, strong arguments support the powerful mathematical thinking

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➤ *Continued from page 11*

associated with mental computation. An example is seen in figuring the cost of three items at \$0.99 each and thinking, that is, knowingly or unknowingly using the distributive property, "3 • 99 is 3 • (100 – 1) or \$3 – 0.03" to determine the cost of \$2.97. Another example is seen in making the connection between finding the product of 19 and 21 mentally and the relationship between $(n - 1)(n + 1)$ and $(n^2 - 1)$.

Estimation received sparse attention in the questionnaire because the questionnaire asked questions with limited value. For example, "using rounding rules to check computation" suggests applying algorithmic rules to estimation. Yet a strong case based on years of both research and practice can be made for encouraging and rewarding flexible rounding, where the actual rounding depends on numbers involved, the operation, and problem context. Conventional rounding rules for estimating the quotient of 3329 ÷ 82 might result in 3000/80 or 3300/80, yet neither of these values is easy to compute. However, 3200/80 or 4000/80 reflects flexible rounding, which produces a reasonable estimate. Flexible rounding, not subject to specific rounding rules, makes messy computations simpler. Finally, estimation is a powerful skill that is not always used to "check computation," as implied in the questionnaire. Indeed many problems can be solved by estimation without resorting to exact computation.

Robert Reys is professor of mathematics education at the University of Missouri in Columbia and codirector of the Show-Me Center Supporting Standards-based Middle Grades Math Curricula.

Narrowing **the Curriculum River**

by Richard M. Oldrieve

For 14 years, I taught students with learning disabilities in the Cleveland Municipal Schools. Every year, salespeople tried to sell me their latest prepackaged curriculum programs. Similarly, when I go to NCTM conventions, I'm confronted by dozens of wonderful sessions where teachers, professors, and curriculum specialists are ready, willing, and able to explain how to teach the newest "constructivist" program.

With such choices, many classroom teachers could have anticipated the summary conclusion of TIMSS: The U.S. mathematics curriculum is "a mile wide and an inch deep." As a proper response to TIMSS, let's redefine elementary school basics back to what every parent, business CEO, and politician truly wants and understands—by the end of the fifth grade, all students will add, subtract, multiply, and divide. Students should demonstrate mastery in stand-alone algorithms, contextual word problems, and naturalistic experiments. Then, where students are thriving, teachers, school districts, and states can decide what other concepts should be taught.

NCTM should lead the charge to a concentrated core curriculum because its members know best that teaching using constructivist pedagogies takes more time. Once the curriculum is loaded with everything but the kitchen sink, directed instruction is the only method that has a prayer of short-term success, and none will have long-term success.

Richard Oldrieve is currently working on his doctorate in literacy education at Kent State University. For the past 14 years, he taught reading and mathematics to elementary school children with learning disabilities in the Cleveland Municipal Schools.

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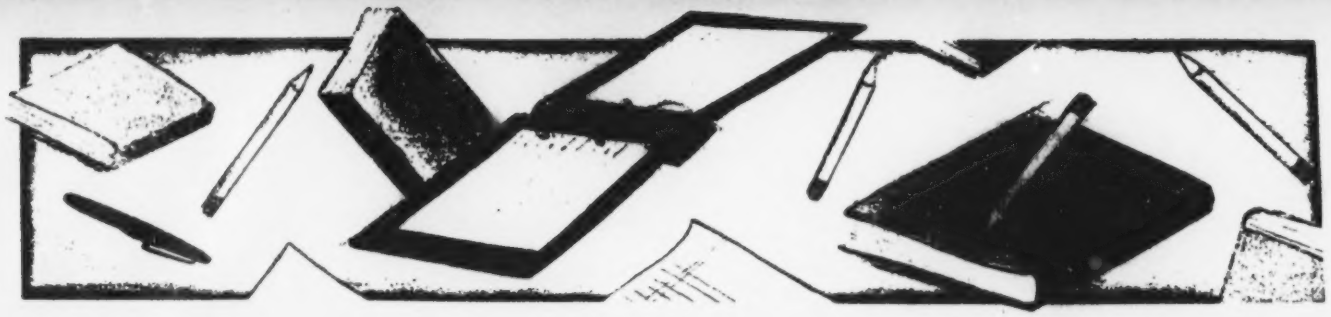

A National Curriculum?

by Cynthia Ballheim, *For the Editorial Panel*

The April 1999 issue of *Dialogues* asked readers whether we should have a national curriculum and why or why not. A total of 475 readers responded to that question. The data show these results: 280—yes, 175—no, and 20—undecided. The reasons for a yes vote included continuity, mobility, and creation of a baseline. The reasons for a no vote included lack of teacher flexibility and creativity and a need for local controls. The undecided group was perplexed. They thought that it would be a good idea, but they were not sure who should determine the national curriculum. Suggestions included teachers, mathematics experts, industry, administrators, parents, scientists, professors, the NCTM, the MAA, the NCSM, departments of education, and a panel of educators from each state. For each reader naming one of these groups, at least two other readers specifically said not to allow the group to create the curriculum!

Samples of reader comments follow:

A national curriculum would freeze an obsolete curriculum in place. The curriculum needs to be dynamic—continually evolving to meet the changing needs of students in a changing world. Only a locally determined curriculum is flexible enough, preferably at the building level....

John Cook
Supervisor, Iowa

Education should prepare every student for meaningful employment. As students move through the system, they must be held accountable. If they do not meet the standards, they do not go on. Alternative programs must be available for those who do not meet the standards.

Lisa Kling
Grades 5–6 Teacher, Ohio

If the national curriculum is established as a guideline for minimum accomplishment, addressing only topics that make up less than 50 percent of any year's taught material, only those who can "pass" their own state's expectations for college sophomores should be allowed significant input; this includes politicians.

Robert N. Baker
Community College Teacher, Alaska

A national curriculum will give many the false security that all students nationwide have access to a good mathematical education.

Andrea Rothbart
University Teacher, Maryland

State control of education is established by Article X of the Bill of Rights.

Lawrence M. Krauser
Grade 8 Teacher, California

Focus will simplify problems now inherent without a national curriculum: political-educational wars over math and splintered textbooks that are inclusive of all content possibly considered.

Janet Doman
Grade 4 Teacher, California

I think that the NCTM Standards are enough.

Barbara McVay
Grades 9–12 Teacher, Texas

Too many schools have lowered expectations for their students' learning primarily because of a K–8 teacher pool that does not know/understand math. A national curriculum would raise expectations for the students in these schools—teachers would have to know more math.

Jeanette N. Pillsbury
Supervisor and Principal K–8, Maine

Lockstep teaching to a minutely prescribed annual curriculum leads to inappropriate divisions of time based on a printed test, not on student need.

Susan Pirie
University Professor, British Columbia

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➤ *Continued from page 13*

Teachers cannot be expected to assimilate all the current research on child development and related mathematics topics. Organizations like the NCTM should take on that role and create a national curriculum.

Mary Jane Stinson
Grades 6–8 Teacher, Vermont

National standards are vital. A national curriculum, however, would reduce the flexibility of schools and districts in addressing variations in resources, ability, and preparation.

Andrew Repp
Grades 9–12 Teacher, Hawaii

I spend too much time on curriculum issues and defending my curriculum to parents. We feed into high-prestige high schools that have different knowledge requirements for acceptance. They, in turn, must satisfy numerous college-entrance requirements. The NCTM is the ideal group to develop national norms.

Ron Zager
Grades K–12 Teacher, Missouri

I teach in Connecticut, which mandates testing in grades 4, 6, 8, and 10. These tests have driven the curriculum. Districts and teachers are judged by the results. However, the tests have never been judged. Many teachers feel that they are inappropriate (timing, some questions, use of calculators) but have no say.

Jo-Ann Olsen
Grade 3 Teacher, Connecticut

We are an extremely mobile society. Without a national curriculum, there will be gaps in students' knowledge through no fault of their own.

Lynette Lindenberger
Grades 9–12 Teacher, Colorado

At the high school level, curriculum could be mandated at a national level. But at the elementary level, the curriculum should be determined by teachers based on students' abilities. The focus should be on teaching children—not on teaching subject areas.

Diane Gower
Supervisor, Tennessee

I am tired of having our nation's education results contrasted with those of countries that have national curricula and being told that we are doing poorly. Why is it that people in our country say that the system of education is performing poorly, but these same people turn down what works in those countries, that is, national curriculum....

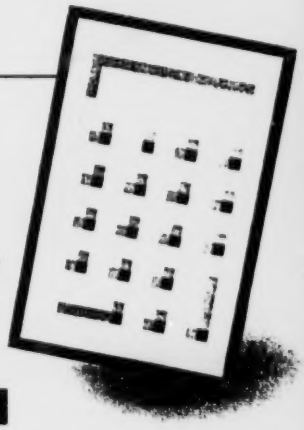
Bill Mortimer
Grades 9–12 Teacher, Washington

Let us say, for the sake of argument, that I agree that we would be better off with a national curriculum.... There will be districts and teachers that do not buy into it.... Unless there is some powerful incentive to follow a national curriculum, we will not have one. This implies that the federal government will need to be involved, establishing a system to ensure that the curriculum is actually being taught.

Corey Andreasen
High School Teacher, Wisconsin

Principles and Standards for School Mathematics must become the framework for a national curriculum. Neither individual teachers, nor schools, nor districts are capable of designing mathematics curricula that can effectively compete with efforts from other countries. We need national leadership. If not NCTM, who? If not a national curriculum, what? If not now, when?

Art Johnson
High School Teacher and Professor, Massachusetts



Our Readers Count on Calculators!

by Cynthia Ballheim, *For the Editorial Panel*

The May/June 1999 issue of *Dialogues* posed the question "Imagine you are teaching the same classes you now teach, but it is the year 2005. On the whole, how do you think your students' amount of calculator usage then will compare with what it is today?" *Dialogues* received 171 responses to this question. The results are tabulated below:

	No Usage	Little Usage	Lots of Usage	Use All the Time
Now	2%	29%	61%	8%
2005	0%	8%	40%	42%

In the year 2005, everyone who expressed an opinion will be using calculators, with 82 percent of the respondents using them a great deal. Ten percent did not respond.

Mathematics Education

Dialogues

Questionnaire

Please respond by 15 November 1999

Please take a few minutes to fill out this questionnaire. Completed forms can be folded, taped, and returned, postage free, to NCTM or faxed to A. Reeves, Editorial Services Unit, at the Headquarters Office at (703) 476-2970. For longer responses, please use regular mail or send e-mail to **dialogues@nctm.org**.

PART 1: *Personal and professional information; please print clearly*

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Home Phone: (___) _____
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Employer: _____
Main Position: _____
❍ K–12 Teacher (Specify grades): _____
❍ Community College
❍ University/College Teacher
❍ Supervisor/Administrator/Consultant
❍ Other: _____

PART 2: *Questions and comments for future issues*

A future issue of *Dialogues* will discuss teacher preparation. We would like some views in advance. Please respond to the following, answering only those questions that are appropriate for you:

❶ Other than the mathematics you teach, what mathematics do you need to know to be an effective teacher?

_____
_____
_____

❷ What changes in your teacher-preparation program would have made you a better mathematics teacher as you began your career? _____
_____
_____

❸ What should mathematics teachers do to stay current? _____
_____
_____

PART 3: *Questions and comments related to the ideas in this issue*

We are interested in your responses to the following questions related to basics.

❶ What mathematics is basic for students to know by the end of the elementary grades?

_____
_____
_____

(Part 3 of this questionnaire continues on the reverse side.)

❷ What mathematics is basic for students to know by the end of middle school?

❸ What mathematics is basic for college-intending students to know by the end of high school?

❹ What mathematics is basic for non-college-intending students to know by the end of high school?

❺ If technology is adequately used to teach mathematics in schools, what topics that once were basic should no longer should be considered basic?

BACK ISSUES OF *DIALOGUES*

Back issues of *Mathematics Education Dialogues* are now available. Remaining stock will be sold in bundles of 25. The cost of the first bundle is $5.00, and additional bundles of 25 cost $2.50. To order, send your check to NCTM at 1906 Association Drive, Reston, VA 20191-9988, or call (800) 235-7566 for credit card orders. Use the stock number shown.

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April 1999	07944D	Who Should Determine What You Teach?
May/June 1999	07995D	Calculators—What Is Their Place in Mathematics Classrooms?

Thank you! The members of the *Mathematics Education Dialogues* Editorial Panel sincerely appreciate your time and interest. Your comments will help improve subsequent issues of *Dialogues*. The Editorial Panel members are Johnny W. Lott, editor; Cynthia Ballheim; Frank Lester; and Paul Zorn.

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